Matrix rings and the Yang-Baxter equation

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The Yang-Baxter equation is an important equation in mathematics and physics. It is relevant to statistical mechanics, quantum information science and numerous other research areas. Following Drinfeld's suggestion, the study of set-theoretic solutions to the Yang-Baxter equation (YBE) have been developed and several interesting connections have been found. In pure mathematics, some of these connections are with braid and Garside groups, regular subgroups and Hopf-Galois extensions, affine manifolds, Knot theory, Hopf algebras, quantum groups, orderability and factorizable groups. In mathematical physics the connections include Yang-Baxter maps, discrete integrable systems, cellular automata, crystals and tropical geometry. Due to this abundance of relationships set-theoretic solutions to the quantum Yang-Baxter equation have been intensively studied.

In 2007, Rump presented some surprising connections between nilpotent rings and set-theoretic solutions of the Yang-Baxter equation. In particular, he showed that every nilpotent ring yields a solution to the Yang-Baxter equation, and every non-degenerate, involutive set-theoretic solution of the Yang-Baxter equation can be obtained from a nilpotent ring, or more generally from a brace (a structure which generalises nilpotent rings). In this talk we look at how to use Rump's method to construct solutions of YBE from sets of nilpotent matrices, and which open problems on nilpotent matrices appear in this context. The set-theoretic reflection equation with the first examples of solutions first appeared in the work of Caudrelier and Zhang.

In this talk we investigate set-theoretic solutions to the Yang-Baxter equation (YBE) and the reflection equation. Amongst other things, we show that for a finite, non-degenerate involutive solution to YBE one only needs to check one of the coordinates to prove that a certain map is a reflection. We give examples of invertible solutions to the reflection equations coming from sets of nilpotent matrices. We also use set-theoretic solutions to construct solutions to the parameter dependent reflection equation (which appears in integrable systems).

For the research topics that we consider in this talk it is sufficient to consider solutions coming from sets of nilpotent matrices. We list some open questions which appear in this context. We also mention connections with other research areas.

This talk will be mainly based on collaborative work with Leandro Vendramin, Robert Weston and Alicja Smoktunowicz.